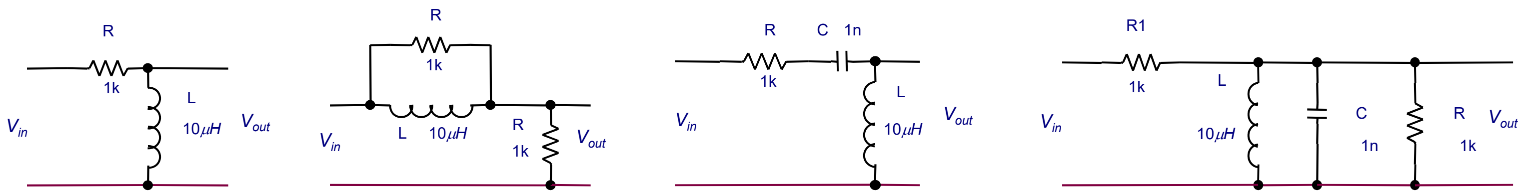
Sinusoidal Response and Discrete Systems

Lecture #5

3CT.2

Homework

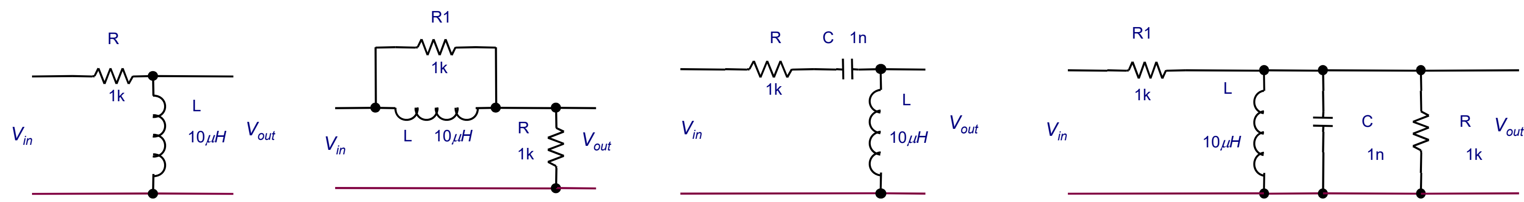
- Sinusoidal Steady State
  - Calculate the Sinusoidal Steady State Response of the network function for the following networks:



- Bode Plots
  - Draw the Bode Plots for these networks.
  - Use Matlab to plot the Bode Plot, submit your code.
- Discrete ODE
  - Calculate the monthly payment Pc
- 3CT.3.1, 3CT.3.2, 3CT.3.4

Homework Answers #1

- Sinusoidal Steady State
  - Calculate the Sinusoidal Steady State Response of the network function for the following networks:

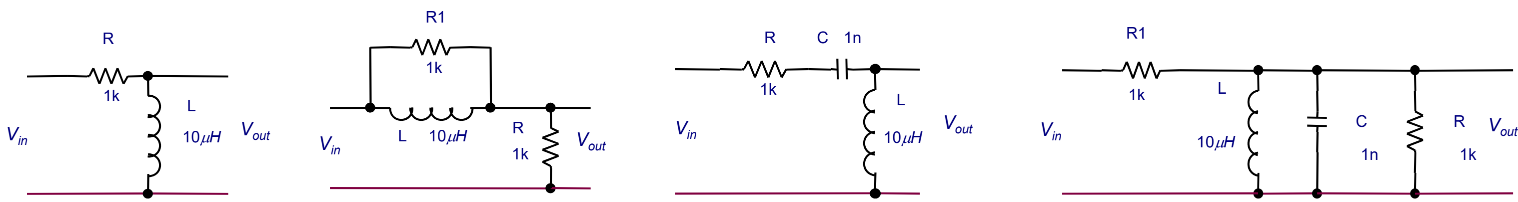


$$a)\ v_{out}(t) = L\frac{di(t)}{dt} = pLi(t); v_{in}(t) = i(t)R + L\frac{di(t)}{dt} = i(t)(R+Lp); \frac{v_{out}(t)}{v_{in}(t)} = \frac{pLi(t)}{i(t)(R+Lp)} = \frac{pL}{(R+Lp)} \Rightarrow \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L}{(R+j\omega L)}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L}{j\omega L + R} = \frac{j\omega 10\times10^{-6}}{j\omega 10\times10^{-6} + 1\times10^{3}} = \frac{j\omega 1\times10^{-8}}{j\omega 1\times10^{-8} + 1} = \frac{\omega 10^{-8}\angle\frac{\pi}{2}}{\sqrt{1+(\omega 10^{-8})^2}\angle\tan^{-1}(\omega 10^{-8})} = \frac{\omega 10^{-8}}{\sqrt{1+(\omega 10^{-8})^2}}\angle\frac{\pi}{2} - \tan^{-1}(\omega 10^{-8})$$

Homework Answers #1

- Sinusoidal Steady State
  - Calculate the Sinusoidal Steady State Response of the network function for the following networks:



$$b)\; v_{out}(t)=i(t)R;\, v_{in}(t)=i(t)R+v_{par}(t);\, v_{par}(t)=i_{R_1}(t)R_1=L\frac{di_L(t)}{dt}=Lpi_L(t);\, i(t)=\frac{v_{par}(t)}{R_1}+\frac{v_{par}(t)}{Lp}\Rightarrow v_{par}(t)=\frac{i(t)}{\frac{1}{R_1}+\frac{1}{Lp}}$$

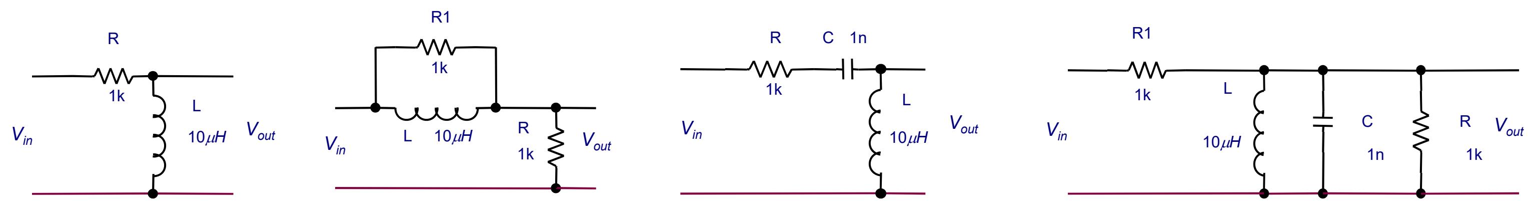
$$v_{in}(t)=i(t)R+v_{par}(t)=i(t)R+\frac{i(t)}{\frac{1}{R_1}+\frac{1}{Lp}};\,\frac{v_{out}(t)}{v_{in}(t)}=\frac{i(t)R}{i(t)(R+\frac{1}{\frac{1}{R_1}+\frac{1}{Lp}})}=\frac{R}{(R+\frac{1}{\frac{1}{R_1}+\frac{1}{Lp}})}\Rightarrow\frac{V_{out}(j\omega)}{V_{in}(j\omega)}=\frac{R}{(R+\frac{1}{\frac{1}{R_1}+\frac{1}{j\omega L}})}$$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)}=\frac{R}{R+R_1\,\|\,j\omega L}=\frac{R}{R+\frac{j\omega LR_1}{R_1+j\omega L}}=\frac{R(R_1+j\omega L)}{R(R_1+j\omega L)+j\omega LR_1}=\frac{RR_1+j\omega LR}{RR_1+j\omega L(R_1+R)}$$

$$=\frac{(1\times10^3)^2+j\omega(10\times10^{-6})(1\times10^3)}{(1\times10^3)^2+j\omega10\times10^{-6}(1\times10^3+1\times10^3)}=\frac{10^6+j\omega10^{-2}}{10^6+j\omega2\times10^{-2}}=\frac{\sqrt{1+(\omega10^{-8})^2}}{\sqrt{1+(\omega2\times10^{-8})^2}}\angle\tan^{-1}(\omega10^{-8})-\tan^{-1}(2\omega10^{-8})$$

Homework Answers #1

- Sinusoidal Steady State
  - Calculate the Sinusoidal Steady State Response of the network function for the following networks:



$$c)\quad v_{out}(t) = L\frac{di(t)}{dt} = pLi(t);\, v_{in}(t) = i(t)R + \frac{1}{C}\int i(t)dt + L\frac{di(t)}{dt} = i(t)R + \frac{1}{pC}i(t) + pLi(t) = i(t)(R+\frac{1}{pC}+pL);$$

$$\frac{v_{out}(t)}{v_{in}(t)} = \frac{pLi(t)}{(R+\frac{1}{pC}+pL)} \Rightarrow \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L}{R+1/j\omega C + j\omega L}$$

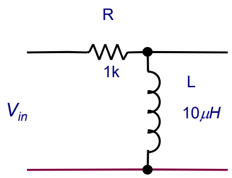
$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L}{R+1/j\omega C + j\omega L} = \frac{-\omega^2 LC}{1-\omega^2 LC + j\omega CR} = \frac{-\omega^2(10\times10^{-6})\times(1\times10^{-9})}{1-\omega^2(10\times10^{-6})\times(1\times10^{-9}) + j\omega(1\times10^{-9})\times1\times10^{3}} = \frac{-\omega^2 10^{-14}}{1-\omega^2 10^{-14} + j\omega 10^{-6}}$$

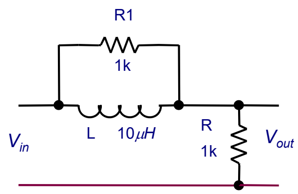
$$= \frac{\omega^2 10^{-14}}{\sqrt{(1-\omega^2 10^{-14})^2 + \omega^2 10^{-12}}}\angle \pi - \tan^{-1}[\omega 10^{-6}/(1-\omega^2 10^{-14})]$$

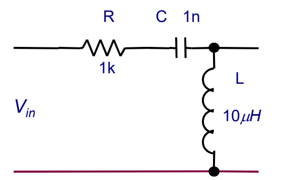
$$\frac{-\omega^2 LC}{j\omega CR} = j\omega\frac{L}{R} = j\frac{1}{\sqrt{LC}}\frac{L}{R} = j\frac{1}{R}\sqrt{\frac{L}{C}} = \frac{10^7 10^{-14}}{10^{-6}} = \frac{10^{-7}}{10^{-6}} = 10^{-1}$$

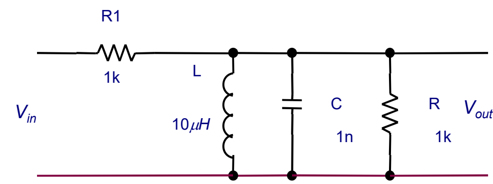
Homework Answers #1

- Sinusoidal Steady State
  - Calculate the Sinusoidal Steady State Response of the network function for the following networks:









$$d)\ \ v_{out}(t) = L\frac{di_L(t)}{dt} = pLi_L(t) = \frac{1}{C}\int i_C(t)dt = \frac{1}{pC}i_C(t) = Ri_R(t); v_{in}(t) = i(t)R_1 + v_{out}(t);$$

$$i(t) = i_L(t) + i_C(t) + i_R(t) = \frac{v_{out}(t)}{pL} + \frac{v_{out}(t)}{1/pC} + \frac{v_{out}(t)}{R} = (\frac{1}{pL} + \frac{1}{1/pC} + \frac{1}{R})v_{out}(t); v_{out}(t) = \frac{i(t)}{\frac{1}{pL} + \frac{1}{1/pC} + \frac{1}{R}}$$

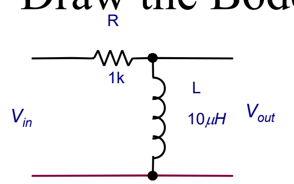
$$v_{in}(t) = i(t)R_1 + \frac{i(t)}{\frac{1}{pL} + \frac{1}{1/pC} + \frac{1}{R}}; \frac{v_{out}(t)}{v_{in}(t)} = \frac{\frac{i(t)}{\frac{1}{pL} + \frac{1}{1/pC} + \frac{1}{R}}}{i(t)R_1 + \frac{i(t)}{\frac{1}{pL} + \frac{1}{1/pC} + \frac{1}{R}}} = \frac{\frac{1}{\frac{1}{pL} + \frac{1}{1/pC} + \frac{1}{R}}}{R_1 + \frac{1}{\frac{1}{pL} + \frac{1}{1/pC} + \frac{1}{R}}} \Rightarrow \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{\frac{1}{\frac{1}{j\omega L} + \frac{1}{1/j\omega C} + \frac{1}{R}}}{R_1 + \frac{1}{\frac{1}{j\omega L} + \frac{1}{1/j\omega C} + \frac{1}{R}}}$$

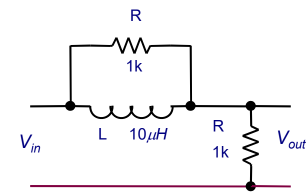
$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{j\omega L \,\|\, 1/j\omega C \,\|\, R}{R_1 + j\omega L \,\|\, 1/j\omega C \,\|\, R} = \frac{\frac{j\omega LR}{R(1-\omega^2 LC) + j\omega L}}{R_1 + \frac{j\omega LR}{R(1-\omega^2 LC) + j\omega L}} = \frac{j\omega LR}{R_1 R(1-\omega^2 LC) + j\omega L(R+R_1)}$$

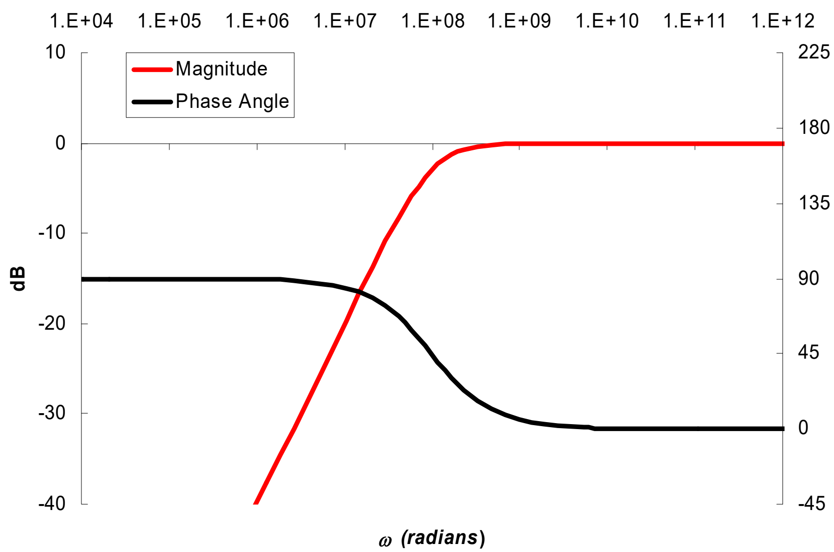
$$= \frac{j\omega 10^{-2}}{10^6(1-\omega^2 10^{-14}) + j\omega 2\times 10^{-2}} = \frac{\omega 10^{-8}}{\sqrt{(1-\omega^2 10^{-14})^2 + \omega^2 4\times 10^{-16}}} \angle \frac{\pi}{2} - \tan^{-1}[\omega 2\times 10^{-8}/(1-\omega^2 10^{-14})]$$

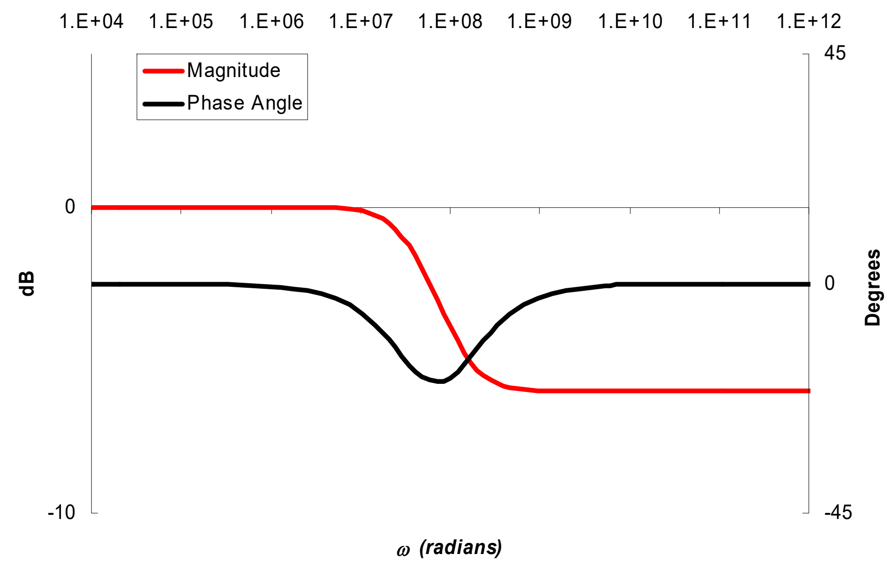
Homework Answers #2

- Bode Plots
  - Draw the Bode Plots for these networks.









$$a)\ \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = H(j\omega) = \frac{\omega 10^{-8}}{\sqrt{1+(\omega 10^{-8})^2}} \angle \frac{\pi}{2} - \tan^{-1}(\omega 10^{-8})$$

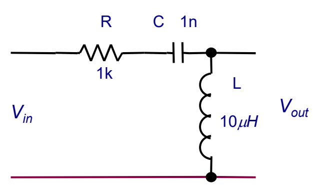
$$H(j0) = 0\angle\frac{\pi}{2};\ H(j10^8) = \frac{1}{\sqrt{2}}\angle\frac{\pi}{4};\ H(j\infty) = 1\angle 0$$

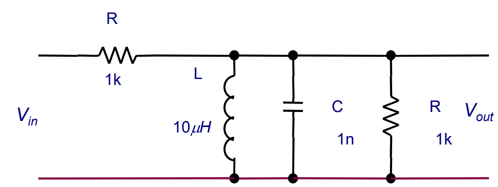
$$b)\ \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = H(j\omega) = \frac{\sqrt{1+(\omega 10^{-8})^2}}{\sqrt{1+(\omega 2\times 10^{-8})^2}} \angle \tan^{-1}(\omega 10^{-8}) - \tan^{-1}(2\omega 10^{-8})$$

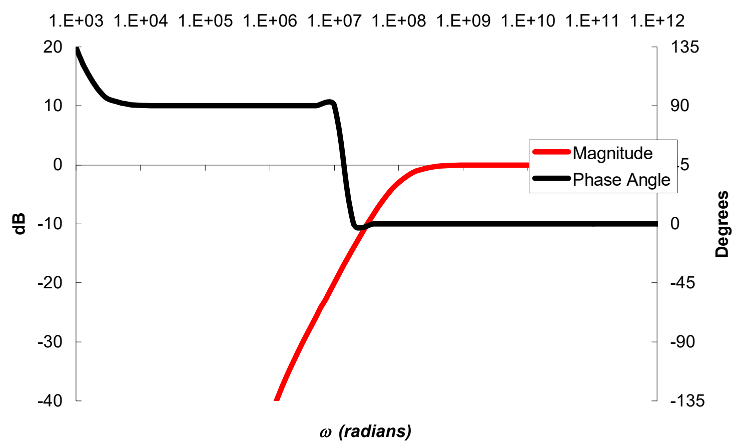
$$H(j0) = 1\angle 0;\ H(j0.5\times 10^8) = 0.79\angle -0.32;\ H(j\infty) = .5\angle 0$$

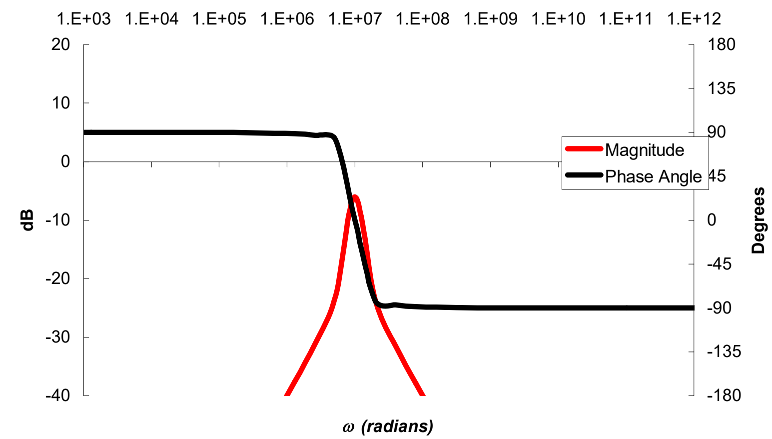
Homework Answers #3

- Bode Plots
  - Draw the Bode Plots for these networks.









c) $H(j\omega)=\dfrac{\omega^2 10^{-14}}{\sqrt{(1-\omega^2 10^{-14})^2+\omega^2 10^{-12}}}\angle\pi-\tan^{-1}[\omega 10^{-6}/(1-\omega^2 10^{-14})]$

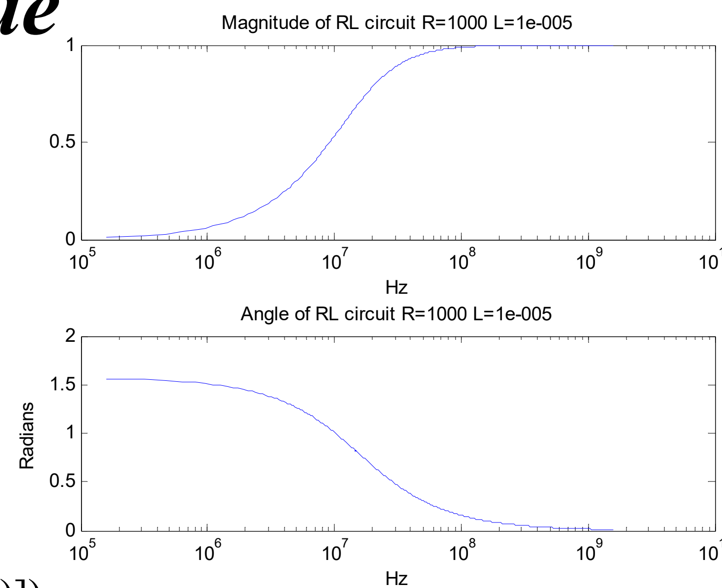
$H(j0)=0\angle\pi;\ H(j10^7)=.1\angle\frac{\pi}{2};\ H(j\infty)=1\angle 0$

d) $H(j\omega)=\dfrac{\omega 10^{-2}}{\sqrt{10^{12}(1-\omega^2 10^{-14})^2+\omega^2 4\times 10^{-4}}}\angle\frac{\pi}{2}-\tan^{-1}[\omega 2\times 10^{-2}/10^6(1-\omega^2 10^{-14})]$

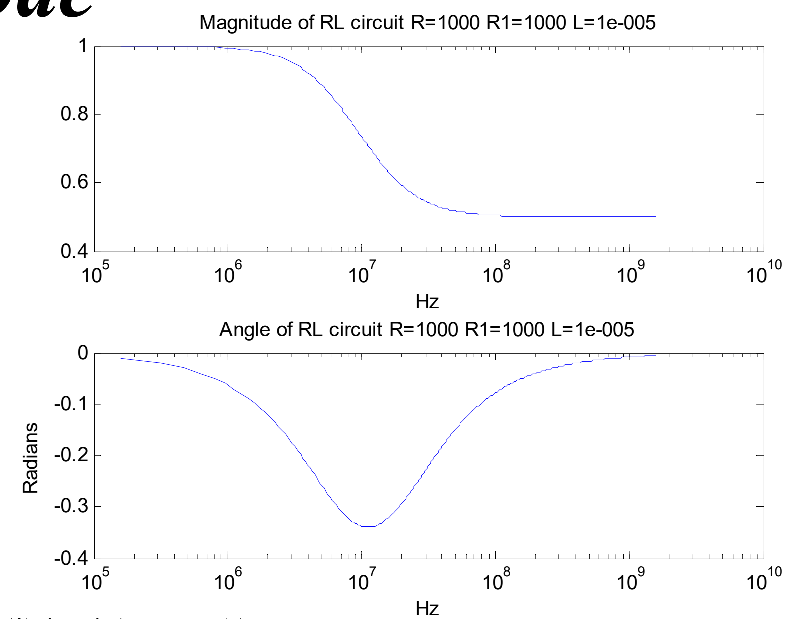
$H(j0)=0\angle\frac{\pi}{2};\ H(j10^7)=.5\angle 0;\ H(j\infty)=0\angle-\frac{\pi}{2}$

Matlab Code

```
clear all;
R=1000;L=1e-5;
omega=(R/L*0.01:R/L*0.01:R/L*100);
maxomega=length(omega);
for j=1:maxomega
    fr(j)=complex(0,omega(j)*L)/complex(R,omega(j)*L);
end
subplot(2,1,1);
semilogx(omega/(2*pi),abs(fr));
title(['Magnitude of RL circuit R=',num2str(R),' L=',num2str(L)]);
xlabel('Hz');
subplot(2,1,2)
semilogx(omega/(2*pi),angle(fr));
title(['Angle of RL circuit R=',num2str(R),' L=',num2str(L)]);
xlabel('Hz');
ylabel('Radians');
```



Matlab Code

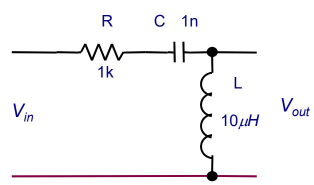


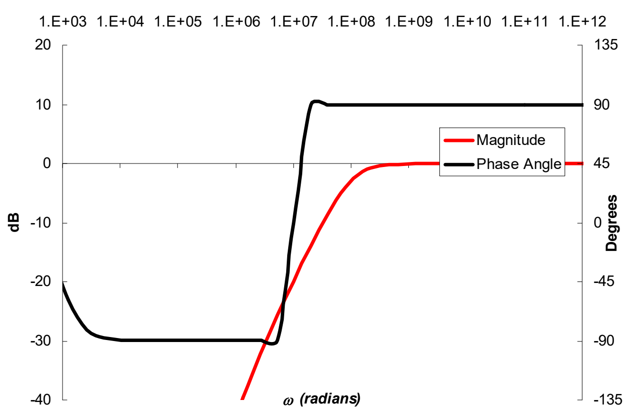

```
clear all;
R1=1000;R=1000;L=1e-5;
wn=R1/L;wd=R1*R/(L*(R1+R));
w=max(wn,wd);
omega=(w*0.01:w*0.01:w*100);
maxomega=length(omega);
for j=1:maxomega
   fr(j)=R*complex(R1,omega(j)*L)/complex(R*R1,omega(j)*L*(R1+R));
end
subplot(2,1,1);
semilogx(omega/(2*pi),abs(fr));
title(['Magnitude of RL circuit R=',num2str(R),' R1=',num2str(R1),' L=',num2str(L)]);
xlabel('Hz');
subplot(2,1,2)
semilogx(omega/(2*pi),angle(fr));
title(['Angle of RL circuit R=',num2str(R),' R1=',num2str(R1),' L=',num2str(L)]);
xlabel('Hz');
ylabel('Radians');
```

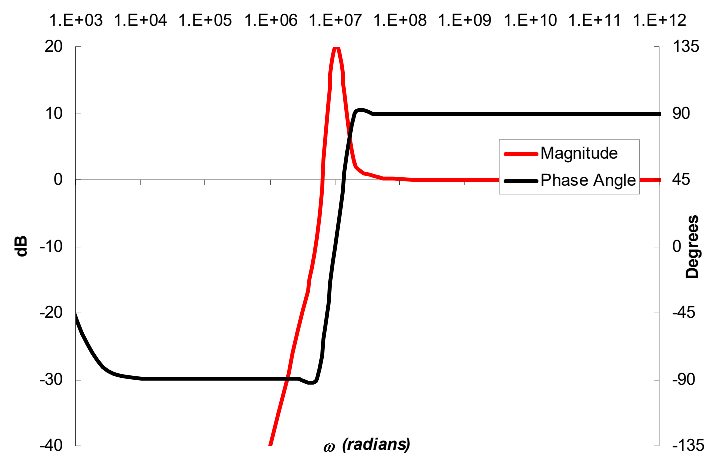
Homework Answers #3a

- Bode Plots
  - Draw the Bode Plots for these networks.

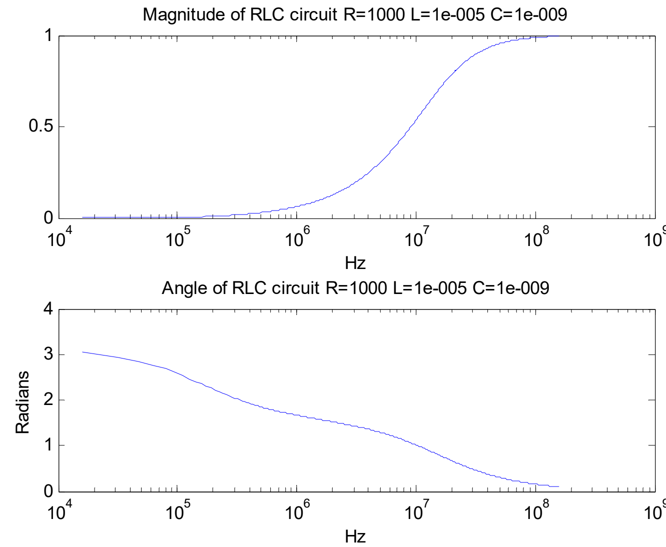
R	Peak	Peak dB
10	10.00	20
100	1.00	0
200	0.50	-6
199	0.50	-6
200	0.50	-6
1000	0.10	-20





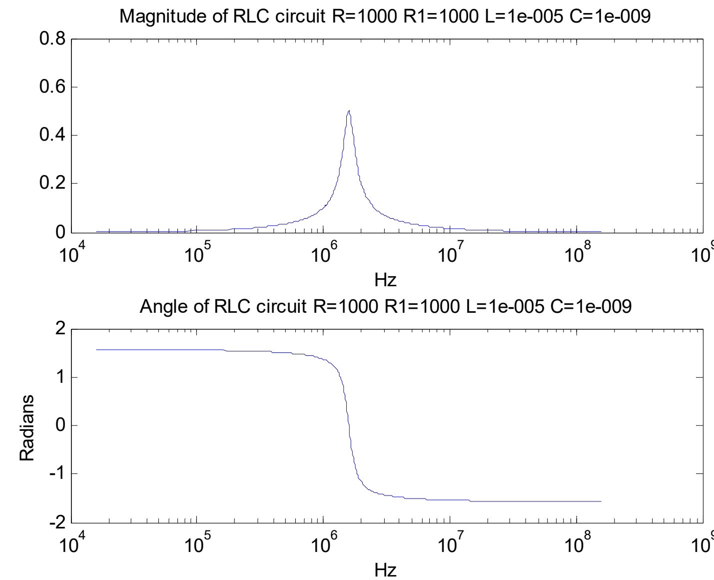


Matlab Code




```
clear all;
R1=1000;R=1000;L=1e-5;C=1e-9;
w=sqrt(1/(L*C));
omega=(w*0.01:w*0.01:w*100);
maxomega=length(omega);
for j=1:maxomega
   fr(j)=complex(0,omega(j)*L)/complex(R,(omega(j)*L-1/(omega(j)*C)));
end
subplot(2,1,1);
semilogx(omega/(2*pi),abs(fr));
title(['Magnitude of RLC circuit R=',num2str(R),' L=',num2str(L),' C=',num2str(C)]);
xlabel('Hz');
subplot(2,1,2)
semilogx(omega/(2*pi),angle(fr));
title(['Angle of RLC circuit R=',num2str(R),' L=',num2str(L),' C=',num2str(C)]);
xlabel('Hz');
ylabel('Radians');
```

Matlab Code




```
clear all;
R1=1000;R=1000;L=1e-5;C=1e-9;
w=sqrt(1/(L*C));
omega=(w*0.01:w*0.01:w*100);
maxomega=length(omega);
for j=1:maxomega
    fr(j)=complex(0,omega(j)*L*R)/complex(R*R1*(1-omega(j)^2*L*C),omega(j)*L*(R1+R));
end
subplot(2,1,1);
semilogx(omega/(2*pi),abs(fr));
title(['Magnitude of RLC circuit R=',num2str(R),' R1=',num2str(R1),' L=',num2str(L),' C=',num2str(C)]);
xlabel('Hz');
subplot(2,1,2)
semilogx(omega/(2*pi),angle(fr));
title(['Angle of RLC circuit R=',num2str(R),' R1=',num2str(R1),' L=',num2str(L),' C=',num2str(C)]);
xlabel('Hz');
ylabel('Radians');
```

Homework Answers #4

- Discrete ODE
  - Calculate the monthly payment Pc

$$(1+r)P(i-1)-P(i)=Pc$$

$$(1+r)\{A_1a^{i-1}+A_2\}-\{A_1a^i+A_2\}=Pc$$

$$\{(1+r)A_1a^{i-1}-A_1a^i\}+(1+r)A_2-A_2=Pc$$

$$1)\ (1+r)A_1a^{i-1}-A_1a^i=0 \Rightarrow a=(1+r)$$

$$2)\ \{(1+\mathrm{r})-1\}A_2=Pc \Rightarrow A_2=\frac{Pc}{r}$$

$$P[i]=A_1(1+r)^i+\frac{Pc}{r};\ P[N]=0 \Rightarrow A_1=-\frac{Pc}{r(1+r)^N}$$

$$\therefore P[i]=\frac{Pc}{r(1+r)^N}\{(1+r)^N-(1+r)^i\}$$

$$P[0]=P_{LOAN}=\frac{Pc}{r(1+r)^N}\{(1+r)^N-(1+r)^0\}$$

$$=\frac{Pc}{r(1+r)^N}\{(1+r)^N-1\}$$

$$Pc=P_{LOAN}\frac{r(1+r)^N}{(1+r)^N-1}$$

3CT.3.1

From 310 chapter 10:

$$H^*(F) = \left(\int_{-\infty}^{\infty} h(t) e^{-j2\pi Ft} dt \right)^* = \int_{-\infty}^{\infty} h(t)^* e^{+j2\pi Ft} dt$$

$$= \int_{-\infty}^{\infty} h(t) e^{-j(-2\pi F)t} dt = H(-F)$$

$h(t)^* = h(t)$ Only is $h(t)$ is real-valued

If $H(F) = Me^{j\psi}$

Then $H(-F) = H^*(F) = (Me^{j\psi})^* = Me^{-j\psi}$

OR

$$H(F) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi Ft} dt$$

$$H(-F) = \int_{-\infty}^{\infty} h(t) e^{j2\pi Ft} dt = H^*(F) = \left(\int_{-\infty}^{\infty} h(t) e^{-j2\pi Ft} dt \right)^* = \int_{-\infty}^{\infty} h(t)^* e^{+j2\pi Ft} dt \text{ since } h(t)^* = h(t)$$

3CT3.2

Again from 310 chapter 10:

$$H(F) = \left|H(F)\right| e^{j\angle H(F)}$$

$$H(-F) = \left|H(-F)\right| e^{j\angle H(-F)}$$

$$\text{Since } H(-F) = H^*(e^{j\hat{\omega}}) = \left|H(F)\right| e^{-j\angle H(F)}$$

Then

$$\left|H(-F)\right| = \left|H(F)\right| \quad \text{even function}$$

$$\angle H(-F) = -\angle H(F) \quad \text{odd function}$$

3CT3.4

$$\frac{d^2y(t)}{dt} + 2\frac{dy(t)}{dt} + y(t) = 3\frac{dx(t)}{dt} + x(t)$$

$$y(t) = Ye^{j2\pi Ft}; x(t) = Xe^{j2\pi Ft}$$

$$\frac{dy(t)}{dt} = (j2\pi F)Ye^{j2\pi Ft}; \frac{dx(t)}{dt} = (j2\pi F)Xe^{j2\pi Ft}$$

$$\frac{d^2y(t)}{dt^2} = (j2\pi F)^2 Ye^{j2\pi Ft}$$

$$(2\pi F)^2 Ye^{j2\pi Ft} + 2(2\pi F)Ye^{j2\pi Ft} + Ye^{j2\pi Ft} = 3(2\pi F)Xe^{j2\pi Ft} + Xe^{j2\pi Ft}$$

$$[(2\pi F)^2 + 2(2\pi F) + 1]Ye^{j2\pi Ft} = [3(2\pi F) + 1]Xe^{j2\pi Ft}$$

$$\frac{Y}{X} = \frac{3(j2\pi F) + 1}{(j2\pi F)^2 + 2(j2\pi F) + 1}$$